Example of the Second - Order Taylor Method

IVP : $\frac{dy}{dx} = 2 x y$

$y(0) = 2$

$y'' = 2 x y' + 2 y$ or $(4 x^2 + 2) y$

Step size : $h = 0.1$

$\frac{1}{2} h^2 = 0.005$

x_0	0	0.1	0.2	0.3	0.4
y_0	2	2.02	2.081004	2.186719	2.343725
y'	0	0.404	0.832402	1.312031	1.874980
y''	4	4.1208	4.494969	5.160657	6.187435
$h y'$	0	0.0404	0.083240	0.131203	0.187498
$\frac{1}{2} h^2 y''$	0.02	0.020604	0.022475	0.025803	0.030937
Δy	0.02	0.061004	0.105715	0.157006	0.218435

$\Delta y = h y' + \frac{1}{2} h^2 y''$

x_0	0.5	0.6	0.7	0.8	0.9
y_0	2.562161	2.856809	3.248763	3.767916	4.456691
y ‘	2.562161	3.428171	4.548269	6.028665	8.022043
y “	7.686482	9.827423	12.865103	17.181696	23.353059
h y ‘	0.256216	0.342817	0.454827	0.602867	0.802204
$\frac{1}{2}$ h^2 y "	0.038432	0.049137	0.064326	0.085908	.116765
Δy	0.294648	0.391954	0.519152	0.688775	0.918970

x_0	1				
y_0	5.375660				
y ‘					
y “					
h y ‘					
$\frac{1}{2}$ h^2 y "					
Δy					

Comparison of the results obtained from the Second - - Order Taylor Method by using various step sizes.

IVP : $y' = 2 \times y \;,\; y(0) = 2$

x	$n=1$ h = 0.1 y	$n=2$ h = 0.05 y	$n=10$ h = 0.01 y	$n=100$ h = 0.001 y	Correct y *
0	2	2	2	2	2
0.1	2.020000	2.020063	2.020098	2.020100	2.020100
0.2	2.081004	2.081439	2.081613	2.081621	2.081622
0.3	2.186719	2.187892	2.188329	2.188348	2.188349
0.4	2.343725	2.346117	2.346983	2.347021	2.347022
0.5	2.562161	2.566448	2.567982	2.568050	2.568051
0.6	2.856809	2.863987	2.866545	2.866658	2.866659
0.7	3.248763	3.260331	3.264449	3.264631	3.264632
0.8	3.767916	3.786163	3.792672	3.792959	3.792962
0.9	4.456691	4.485167	4.495361	4.495811	4.496816
1	5.375660	5.419929	5.435851	5.436556	5.436564

* $y = 2\, e^{x^2}$